How to be a Celestial Mechanic Part I

Following in Herschel's Footsteps
by John Jardine Goss

Friedrich Wilhelm Herschel had a plan that would answer the biggest astronomical question of his day: How large is the universe? He believed that the small but measurable shift in stellar positions as viewed from opposite sides of the earth's orbit, known as "parallax," would enable him to calculate the approximate distance to the stars. However, shifts of this nature would be only on the order of a few arc seconds. Most certainly, it was going to be a very difficult project, requiring numerous exacting measurements.

It was known that a good number of stars happened to be visually close to another star. Many astronomers in the late 18th Century believed that these neighbors were mutually gravitationally associated Doppelsterne, or double stars. Herschel, on the other hand, thought they were just chance line-of-sight viewings, with one star being much farther from our sun than the other. If that was indeed the case, then the closer of the pair may exhibit a distinguishable parallax. And its degree could easily be measured by comparing their apparent separation 180 days apart. In Herschel's mind, all that needed to be done was to firmly establish the position of as many double stars as possible. He never did find the distance to the stars, but his efforts hardly proved fruitless...

On an English March night 219 years ago, the known solar system was about to double its diameter. William Herschel was scanning the dense star fields near the Taurus-Gemini border, inventorying all he saw. Observational astronomy was still in its infancy and few sections of the sky had been thoroughly cataloged. While looking for double stars, he wrote,

> On Tuesday, the 13th of March, between ten and eleven in the evening, while I was examining the small stars in the Neighborhood of H Geminorum, I perceived one that appeared visibly larger than the rest; being struck with its uncommon magnitude, I compared it to H Geminorum and the small star in the quartile between Auriga and Gemini, and finding it so much larger than either of them, suspected it to be a comet. ...The power I had on when I first saw the comet was 227.

He followed his new "comet" for the next several weeks, carefully recording its position among the stars. By March 19th he concluded that it followed the ecliptic very closely and had an hourly motion of 2 1/4 minutes. This "comet" was not acting like a comet at all. Herschel's notes from April 6th stated, "With a magnifying power of 278 times, the Comet appeared perfectly sharp upon the edges, and extremely well defined, without the least appearance of any beard or tail." By April 29th Charles Messier had written to Herschel about his discovery, adding that, "nothing could be more difficult than to recognize the new comet, and I cannot conceive how you were able to return several times to this star-or comet... since it had none of the characteristics of a comet."

During this time, others computed the distance and orbital motion of the new body. It turned out to be 19 Astronomical Units from the sun and moving in a nearly circular orbit with a period of 82 years. (An "Astronomical Unit," or "AU," is the average distance of the Earth from the

Sun-about 93,000,000 miles.) As a rule, comets do not appear perfectly sharp, and do not move in such an orbit, but planets do. By May, it became clear that this was a planet--a planet that an Englishman had discovered. Actually, Herschel was from Germany. But he had emigrated to England and was living an English life. Britain may have just lost the Colonies, but it gained a planet! Herchel's new world soon became known as Uranus.

How to be a Celestial Mechanic Part II

The Solar System's Distant Outpost
by John Jardine Goss

Part two of a mini-series about Herschel's discovery of Uranus and how amateur astronomers can replicate the observations and calculations today.

How did Herschel and his contemporaries calculate the new planet's orbit? How did they know it was about 19 Astronomical Units from the sun and followed a nearly circular orbit of 82 years? These observations and calculations are not out of reach of amateur astronomers today. After all, Herschel's telescope of choice was his homemade 6½ inch f/14 Newtonian reflector. In the 219 years since its discovery, Uranus has become another interesting telescopic sight for amateurs.

Another of Herschel's homemade instruments was an eyepiece containing fine parallel hairs of a known apparent angular separation. This allowed him to judge the distance between two close objects. During the weeks after his initial discovery, Herschel measured the mysterious body's movement among the background stars. After a few weeks of many meticulous observations, he concluded that the body moved about 43 arc seconds per day.

By the time of Herschel, celestial mechanics had been advancing for 180 years since Kepler formulated his laws of planetary motion. Kepler's third law, the Harmonic Law, provided a starting point for calculating orbits. It is commonly expressed as:

$$p^2 = k(a^3)$$

where "p" is the orbital period stated in years, "a" is the mean distance from the sun stated in Astronomical Units and k is a proportionality constant with the value of 1.

When Herschel first found Uranus, its heliocentric longitude (i.e. its position along the ecliptic as seen from the viewpoint of the sun) was 88 degrees 18 arc minutes. By the time Charles Messier congratulated him on his new comet, its longitude was 88 degrees 52 arc minutes. This movement of 34 arc minutes in 47 days equates to 4.50 degrees/year which is the annual motion of Uranus in the sky. Its orbital period, p, is found by dividing its annual motion into 360 degrees, which results in 80.01 years. Inserting that number into the Third Law:

$$(80.01\text{years})^2 = 1(a^3)$$

After rearranging the equation and solving for "a," it is found that its mean solar distance is 18.57 AU.

Calculations using different observation times will eventually show that the planet is not always at 18.57 AU from the sun. Using its observed coordinates and calculated solar distance, it will be seen that Uranus does trace a very slight elliptical orbit.

Next Month: "Determining Uranus' Orbit Today," the third and final installment of the series, which provides step-by-step instructions for completing the observations and calculations.

How to be a Celestial Mechanic Part III

Determining Uranus' Orbit Today
by John Jardine Goss

The third and final installment of a mini-series about William Herschel's discovery of Uranus and how amateur astronomers can replicate the observations and calculations today.

Amateur astronomers can successfully determine Uranus' orbit today by carefully measuring its position among the background stars. Because the earth is always moving along its path around the sun, Uranus' true movement is most apparent when the sun-earth-Uranus angle is 90 degrees (i.e. at western and eastern quadrature). At this time, the earth is moving directly towards or directly away from Uranus, and has little or no lateral movement that can confuse the interpretation of positional measurements. The quadratures occur 3 months before and after Uranus' opposition with the sun as seen from the earth. Since its next opposition is August 11, 2000, the best time for orbital observations this year are the 2 weeks centered around either May 11th or November 9th. An observing program could be accomplished by following a few simple steps.

The eyepiece selected should have an actual field of view of at least 20 arc minutes. Uranus should remain in this region for 27 days, thereby allowing sufficient time to conduct all the necessary observations. The true field of view can be found by using the equation:

$$FOV = (15)(t)(\cos dec)$$

where:

FOV is the true field of view expressed in decimal minutes,
t is the time that it takes a test star to drift across the full field of the eyepiece,
dec is the declination of the test star expressed in decimal degrees.

As accurately as possible, plot the position of Uranus among the background stars. Draw everything that is in the field of view of the eyepiece. Note the date as well as the time of day.

A few days later, again plot the planet's position with respect to the field stars on the chart. Since Uranus moves about 43 arc seconds per day, there should be quite a change in its location. Again, note the date and time of day. Continue to do this for a few weeks.

With a ruler, measure the distance the planet moved between each observation. Since the linear diameter of the field of view and its angular measure are both known, the true angular distance between the observations can be determined using basic proportions.

Finally, compute Uranus' movement in arc seconds per day. As stated in step 3, this value should be around 43. Determine Uranus' annual movement which should be expressed in degrees. Then calculate how many years it will take the planet to travel once around the sun (or in other words, to travel 360 degrees in its orbit). This last value should be nearly 84 years.

Insert the value obtained in step 5 into the equation for Kepler's Harmonic Law:

$$(84)^2 = 1(a^3)$$

Solve for "a," the semi-major axis of the planet's orbit (in other words, its distance from the sun). The value should be about 19 AU.

If measurements are taken for more than a couple of weeks, the effect of the earth's orbit will begin to be seen. Now, determining Uranus' orbit will involve manipulating a real trigonometric mess!

This would make a very impressive project for a student in a high school science class. After all, how many people have measured the distances to the planets; in this case, a span of nearly two billion miles? This elementary method also can be used to determine the orbits of the other three gas giants. Because of their relatively "close" proximity to the earth, the window of observing opportunity for Jupiter and Saturn will be approximately 1 week (again around western or eastern quadrature) as compared to Uranus' 2 weeks. Neptune will pose a challenge in locating its small 7th magnitude, 3 arc second diameter disk. But once it's located and a few measurements are taken, it will be seen that its distance is nearly 3 billion miles away! Truly amazing results especially when considering that the readings were taken using such a small telescope! This is how a Celestial Mechanic is born!